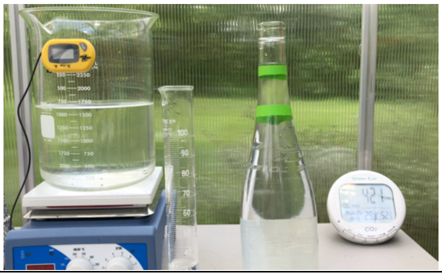
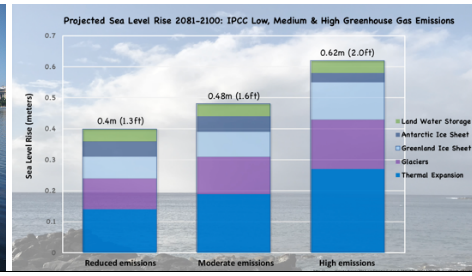
THERMAL EXPANSION OF WATER





Overview

We observe thermal expansion every time we use a simple liquid-filled thermometer. Changing the temperature of a material changes its density and also its volume. In a thermometer this forces the liquid to expand into a narrow tube, which is calibrated to show us the temperature. The same property applies to larger systems – like the water in the ocean – and is one of the principal concerns of climate change. There is a ***coefficient of thermal expansion*** that is unique to each material that determines how much the material will change volume when heated. In this activity we calculate the coefficient of thermal expansion for tap water.

- Introduction
  - Students will heat water in a long-necked glass bottle to explore the relationship between temperature and volume of water. Quantifying the initial volume, change in volume, and the initial and final temperatures allows students to calculate the coefficient of thermal expansion for tap water.

- Grade Level
  - This activity can be completed by students from grades 8-14. The calculation requires the use of intro-level algebra, although younger students can perform the experiment non-quantitatively.

- Student Learning Objectives
  - Students will set up and conduct a quantitative experiment
  - Students will measure and record changes in a simple system
  - Students will use algebraic manipulation of their own data to calculate the coefficient of thermal expansion for water
  - Students will explore the relationship between temperature and volume changes in a liquid, and apply their experimental results to real-world systems

- Lesson Format
  - In School: This is a lab activity. Ideally students will work in small groups – although if the availability of equipment constrains group size, the experment can be done as a class demonstration.
  - Virtual Lab: With access to a few basic tools this activity can be done in a remote learning environment using the Youtube video for guidance: https://youtu.be/7q2SGL_qmbg
- Time Required
  - This activity requires one or two lab periods; the experiment itself will take one period, while discussion and analysis will require an additional class period.
- Education Standards
  - NGSS Planning and Carrying Out Investigation
  - NGSS Analyzing and Interpreting Data
  - NGSS PS1.A: Structure & Properties of Matter
  - ESS3.C: Human Impacts on Earth Systems
  - ESS3.D: Global Climate Change
  - NGSS Scale, Proportion & Quantity
  - NGSS Energy & Matter
  - Grade Level: Middle/High School
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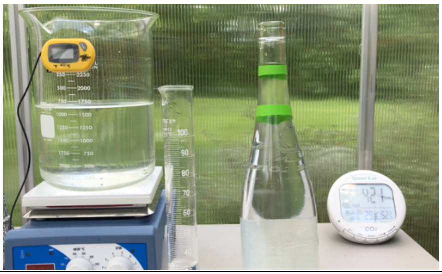
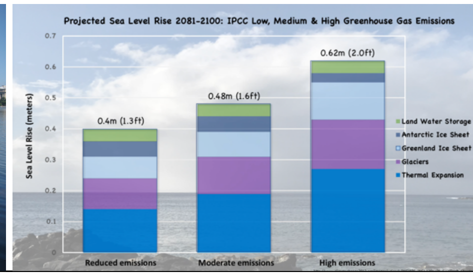




Instructions & Materials

Resources

- Video | In the Greenhouse #6 | Thermal Expansion & Sea Level Rise
  YouTube: https://youtu.be/7q2SGL_qmbg
- Instructor Handout
- Student Handout

In-school Activity

- Download the Instructor Handout & Student Handout

The Instructor Handout duplicates all the content of the Student Handout, with additional context and directions for setting up and running the experiement.

- Set up and run the experiment yourself first, to gauge the range of behavior your students are likely to encounter. If you are limited by time or equipment – for example not enough hot plates for students to work in small groups, or short class periods – you can have one experimental set-up (saving time if you've set it up ahead) and have each team work independently with the group data. Decide what type of additional activities (if any) you would like the class to pursue.

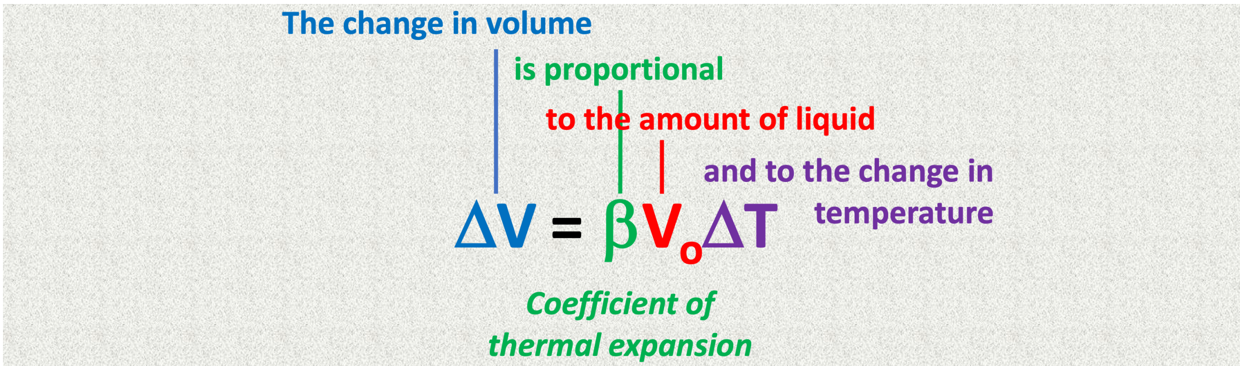
Materials (duplicate this list for each student group)

- Hot plate, stove top, or similar, to heat a pan of water
- Long-necked bottle (ex: soda, wine)
- Graduated cylinder/beaker/measuring cup, to measure volume of water in bottle
- Small grad cylinder/beaker/measuring cup to measure volume change
- Large beaker or sauce pan, to keep bottle from direct contact with stove/hot plate
- Thermometer – metal-stem digital kitchen thermometer or digital aquarium thermometer
- Masking tape
- Calculator or computer spread sheet

Instructions

This experiment asks students to heat water in a bottle, a process that forces the warm water to expand into the bottle neck. Students measure the temperature change and volume change of the water, and use these values to calculate the coefficient of thermal expansion for tap water at room temperature. Once students have gained this expertise they can apply their knowledge to larger and more important systems; notably the projected sea level rise due to warming of the global oceans.

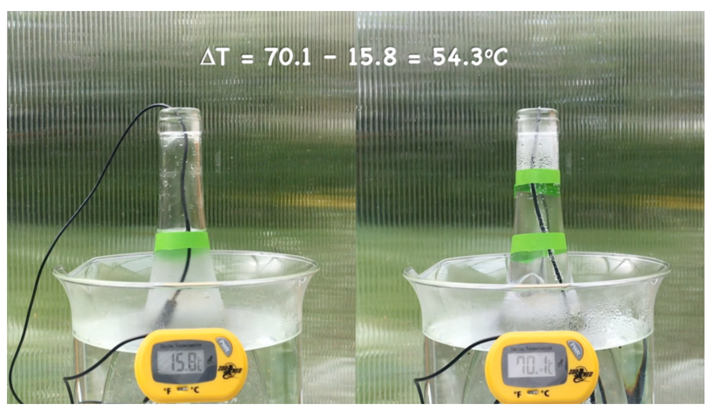
When a material is heated, its change in volume is proportional to the amount of material and to the change in temperature. That idea can be expressed mathematically,



where the constant of proportionality is the coefficient of thermal expansion, commonly designated with the Greek letter **β** (sometimes α), and with units of K^{-1}, or "per degree." The initial volume is designated V_o, the volume change is ΔV and the temperature change is ΔT.

[Note: this is a nice example that supports math literacy in that the verbal description of the concept translates directly word-for-word to the mathematical expression.]

The step-by-step instructions for setting up the experiment are given in the teacher and student handouts [links]



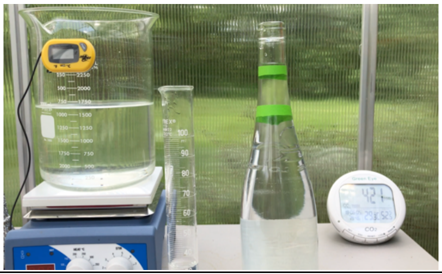
Students will add a known volume of water to the long-necked bottle – enough to bring the water level to the bottom of the narrow neck. A thermometer is inserted into the bottle, and students mark the initial water level on the bottle and record the initial temperature. The bottle is placed in a larger heat-resistant water-filled container (to keep the bottle from direct contact with the heating element and prevent breakage) and the entire system is heated through a temperature change of ca. 40℃ (this will take appx 10-20 minutes, depending on the power of the heating element). Students record the final temperature and remove the bottle from the water bath to mark the final volume on the bottle neck. At this point students will design a protocol for measuring the change in volume. Since they began with cold/room temp water they will need to re-measure the new volume with water at the initial temperature, not the final warmer temperature. One method is to empty the warm water and re-fill the bottle to the new volume mark with cold water, then pour the new volume of water into a beaker/graduated cylinder, subtract the new volume from the initial volume to find the volume change. Data can be recorded on a table like the one below. Note that the boxes with green shading are those that require a calculation whereas unshaded boxes are direct measurements.

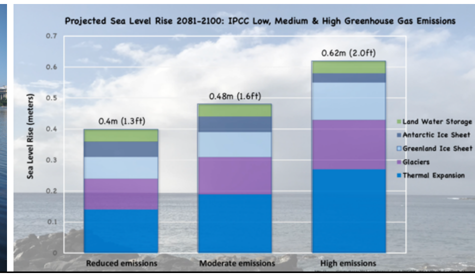
Data Table

	Volume (liters)	Temperature (C)	Coefficient of Expansion (T^{-1})
Initial Volume (Vo)	V_o=		
Volume change (ΔV)	ΔV=		
Initial Temp (T_1)		T_1=	
Final Temp (T_2)		T_2=	
Temp Change (ΔT)		ΔT=	
Coefficient of Expansion (β)			β=

Solution

The coefficient of thermal expansion for water in this experiment should be $\boldsymbol{\beta}$~0.0004/C.





Background & Extensions

Overview
The coefficient of thermal expansion is not a universal constant. It varies as a function of the materials that we might examine, and for each material the coefficient varies as a function of the ambient temperature and pressure. For example, the coefficient of thermal expansion of the iron in the Eiffel Tower is different from that of fresh water, and the coefficient of thermal expansion is different for fresh water and for salt water.

Photo: The Eiffel Tower in Paris is taller in the summer than in the winter due to the thermal expansion of its iron structure. Typical annual temperature variations cause the tower to increase in height by about 15cm (6”) in the summer as opposed to winter. Also, because the sun shines on one side at a time, the sunny side also expands with respect to the three shady sides, causing the tower to tilt, so that the summit actually moves in a circle (diameter ~15cm) in opposition to the movement of the sun over the course of a day. (https://www.toureiffel.paris/en/news/history-and-culture/why-does-eiffel-tower-change-size)

The temperature variation for the volumetric coefficient of expansion for fresh water and sea water is shown in the table below:

Volumetric Expansion Coefficients for Fresh Water and Sea Water

Temperature ℃	Fresh Water β (/℃)	Sea Water β (/℃)
0 ℃	-0.000050 /℃	0.000051
4	0	0.000101
10	0.000088	0.000167
20	0.000207	0.000257
30	0.000303	0.000334
40	0.000385	
50	0.000457	
60	0.000522	
70	0.000582	
80	0.000640	
90	0.000695	

Clearly thermal expansion is a sensitive parameter – and it might seem like a messy quantity to introduce to students – but it is also a critically important one. And because it is so important, materials scientists and engineers have worked for centuries to figure out expansion coefficients for every possible material in every possible environment; thus these are quantities that we know ***very*** well. A walk across any bridge should reveal the expansion joints engineered into the construction to accommodate seasonal and diurnal temperature changes, and the expansion gaps between the steel rails on a railroad track give rise to the characteristic "clickety-clack" sound of moving trains. Thermal expansion in the built environment is all around us, even if we don't think a lot about it.

Thermal expansion in the natural environment is equally important, and in the 21st century the thermal expansion of sea water is a parameter that will impact millions of people globally. More on that below.

Tips & Tricks

Safety first. In the video I heated the water to 70℃, which is scalding hot. The experiment will work just as well at lower temperatures. 50℃ = 120°F, which is a safer temperature for inattentive students. Since both the temperature change and the initial volume of the water determine the amount of volume change, using a larger bottle – a wine bottle as opposed to a soda bottle – will produce a larger and more visible increase in volume, especially if you choose to heat the water less.

A digital thermometer is always easier to read than an analog thermometer. In this experiment the thermometer itself takes up some of the volume of the bottle, so it is important to make sure that the initial volume and final volume are recorded under the same conditions – either with the thermometer in, or out, of the bottle. An aquarium thermometer is ideal for this experiment as it is easy to use with the bottle and has a big easily-read display.

To calculate the coefficient of thermal expansion from the equation given here requires a simple algebraic manipulation. For students who do not have this background the equation can be re-arranged ahead. If students wish to compare their result with published values for the coefficient of thermal expansion they will need to consider the temperature range over which they heated the water, since β changes as a function of temperature. For example, if the water was heated from 20 to 60°C, the appropriate temperature to compare to is the midpoint of that range, 40°C, where $\beta=0.000385$.

One final consideration; engineers tabulate the coefficients of thermal expansion for both volume expansion and linear expansion (ex: a piece of steel re-bar experiences mostly linear expansion). In this instance we are measuring the volumetric expansion of water, even though we will think about consequences such as sea level rise, which is a linear measurement. More below.

Take it Beyond the Classroom

In this experiment we use a very small volume of water and heat it over a very large temperature range. In the natural world it is more common for a large volume of water to be heated over a very small temperature range. The goal of this experiment is to give students experience with the concept of thermal expansion so that they can consider systems that are beyond their ability to measure directly. The most important such system is the change in volume of ocean water in response to changes in global temperature.

Data for the Ocean

Surface area	3.619×10^{14} m^2
Total volume	1.335×10^{18} m^3

Table 1. Volumes of the World's Oceans from ETOPO1

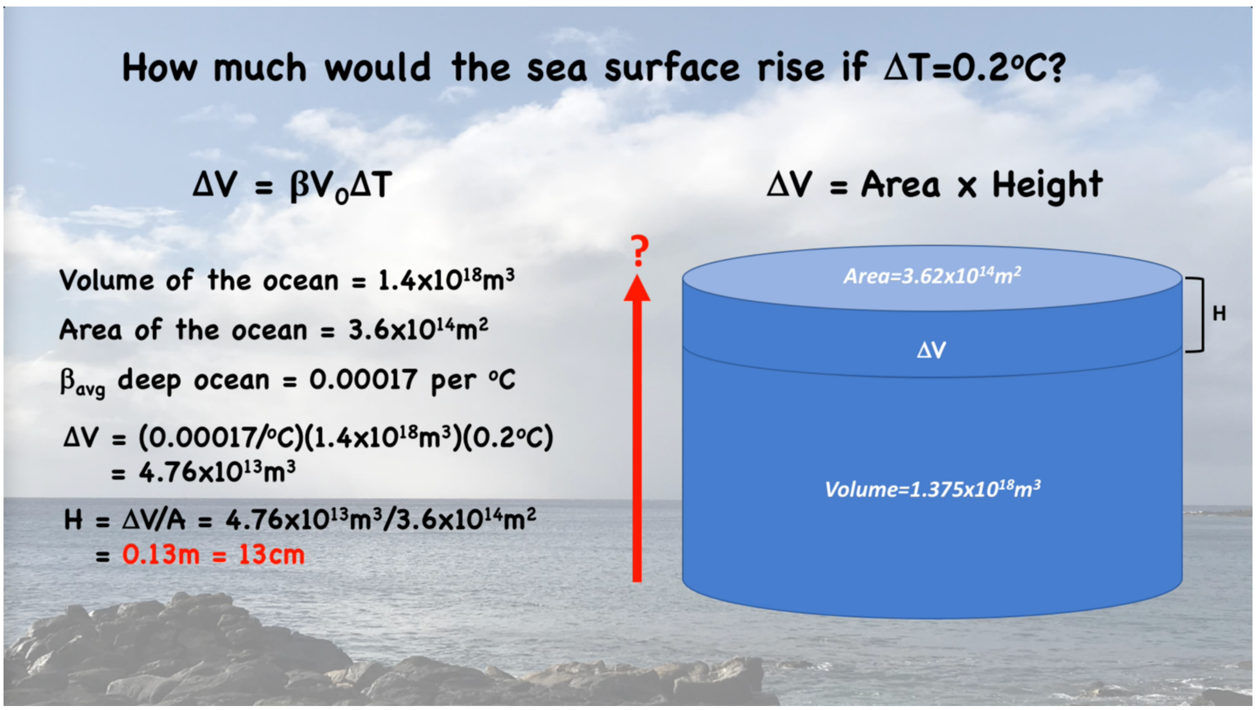
Download Table 1 as a .PDF

	Area+ (km2)	% Ocean Area	Volume (km^3)	% Ocean Volume	Avg. Depth (m)	Max Depth (m)
Arctic Ocean	15,558,000	4.3	18,750,000	1.4	1205	5567
Atlantic Ocean	85,133,000	23.5	310,410,900	23.3	3646	8486
Baltic Sea	***406,000***	***0.1***	***20,900***	***0.0***	***51***	***392***
Mediterranean	***2,967,000***	***0.8***	***4,390,000***	***0.3***	***1480***	***5139***
North Atlantic	***41,490,000***	***11.5***	***146,000,000***	***10.9***	***3519***	***8486***
South Atlantic	***40,270,000***	***11.1***	***160,000,000***	***12.0***	***3973***	***8240***
Indian Ocean	70,560,000	19.5	264,000,000	19.8	3741	7906
Pacific Ocean	161,760,000	44.7	660,000,000	49.4	4080	10,803
North Pacific	***77,010,000***	***21.3***	***331,000,000***	***24.8***	***4298***	***10,803#***
South Pacific	***84,750,000***	***23.4***	***329,000,000***	***24.6***	***3882***	***10,753***
South China Sea	6,963,000	1.9	9,880,000	0.7	1419	7352
Southern Ocean*	21,960,000	6.1	71,800,000	5.4	3270	7075
Total:	**361,900,000¤**	**100.0**	**1,335,000,000**	**100.0**	**3688**	**10,803**
error estimates	0.10%		1%			

https://www.ngdc.noaa.gov/mgg/global/etopo1_ocean_volumes.html

In its 2021 Assessment Report (IPCC AR6) the Intergovernmental Panel on Climate Change presented five climate change scenarios, each based on a range of assumptions about anthropogenic greenhouse gas emissions over the next century. The scenarios range from very-low and low emissions through intermediate, high, and very high emissions. Each set of greenhouse gas emissions drives a temperature increase and subsequent sea level increase (along with other outcomes not discussed here, but which can be found in the AR6 Report (https://www.ipcc.ch/report/ar6/wg1/#SPM).

We can select representative temperature increases for sea water and use an appropriate coefficient of thermal expansion for the ocean to calculate the contribution to sea level rise from the thermal expansion of sea water. The calculation for a 0.2°C increase in the temperature of the ocean is shown in the figure below.



Sea Level Rise for Projected Ocean Warming Scenarios

ΔTemperature (°C)	ΔVolume (m^3)	ΔSea Level (m)
0.2	4.54×10^{13}	0.13
0.3	6.81×10^{13}	0.19
0.4	9.08×10^{13}	0.25

When projecting global sea level rise, thermal expansion accounts for just under half of the rise, while water inputs from melting ice caps and glaciers is a little more than half. So the results in the table above represent total sea level increases from 0.26-0.50 meters, ca. 1-2 feet. These results are well-aligned with the IPCC scenarios shown in the figure below. Additional discussion of climate, oceans and ice is found in the IPCC Special Report on the Ocean and Cryosphere in a Changing Climate (https://www.ipcc.ch/srocc/).

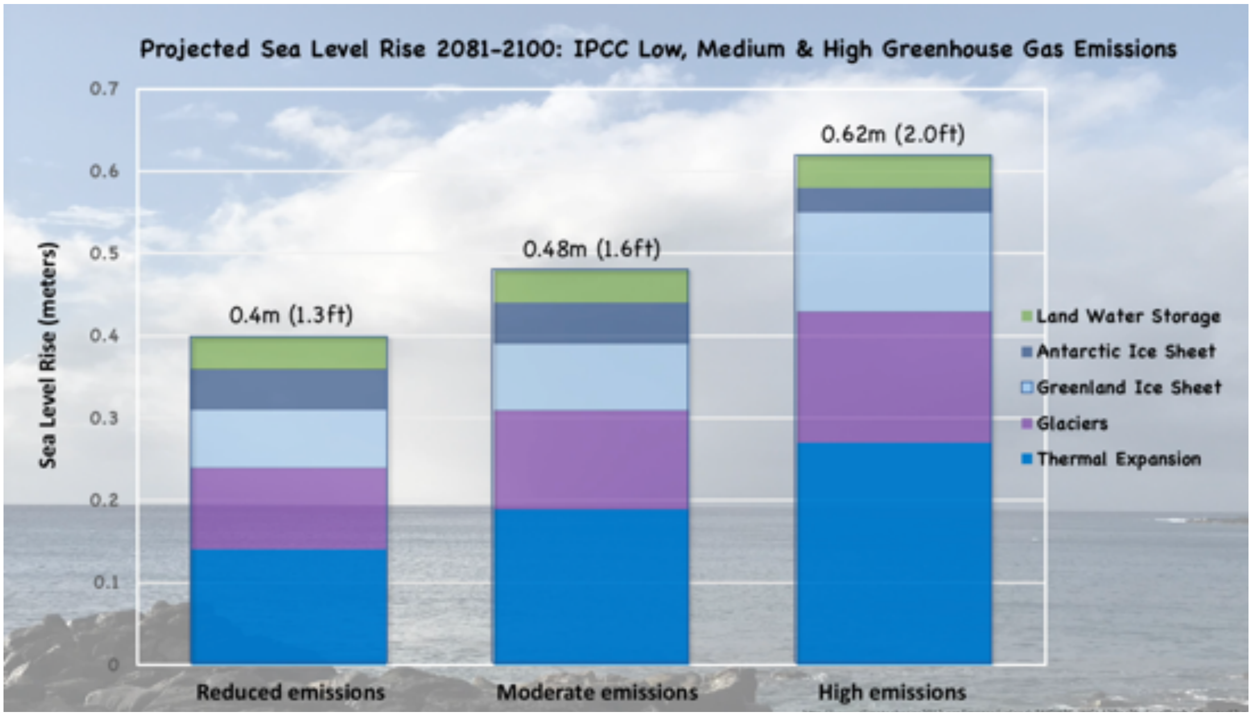
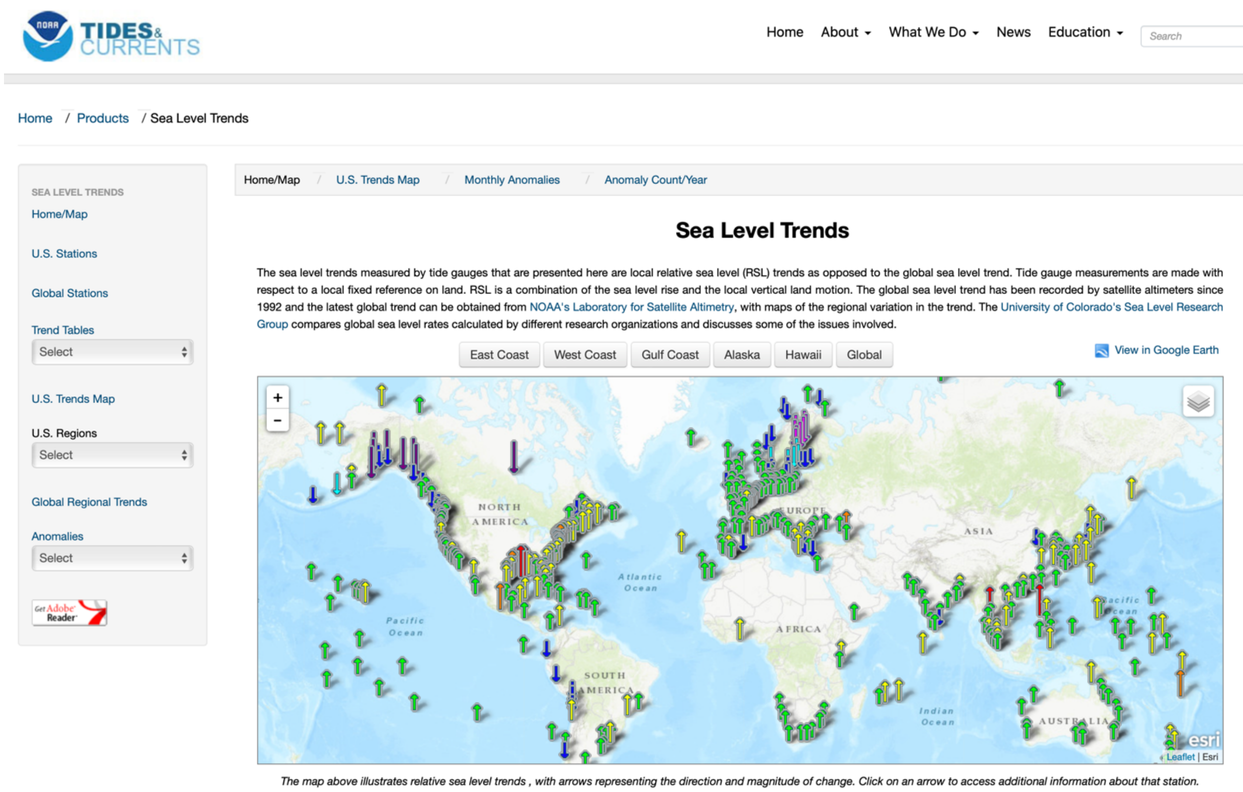


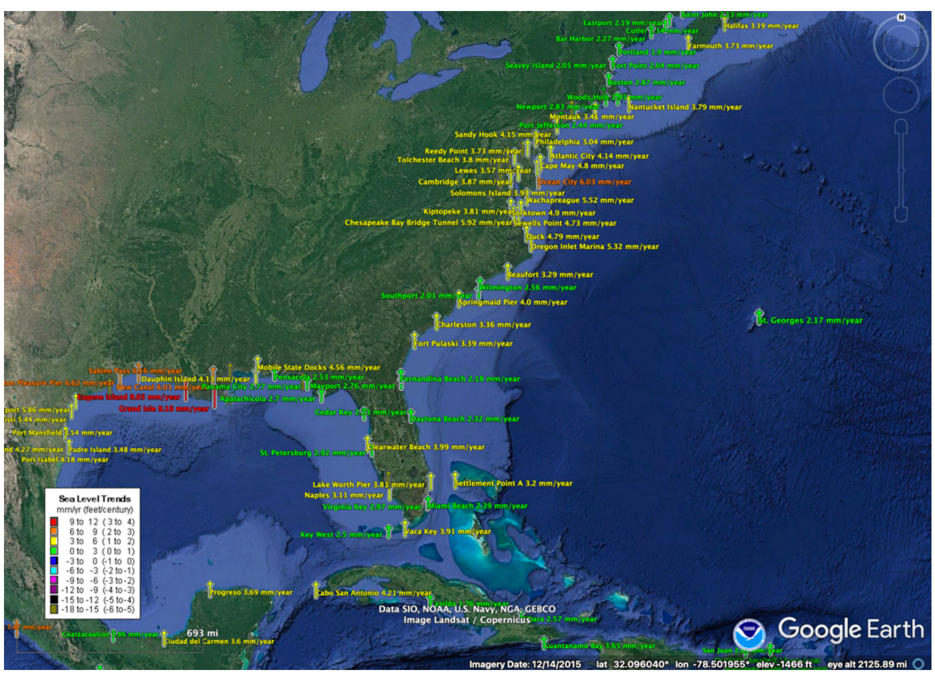
Figure: Results of three emissions scenarios from the IPCC Special Report on the Ocean and Cryosphere in a Changing Climate

Why does this matter? The sea level rise projections might appear to be small, but they are in fact significant, for several reasons:

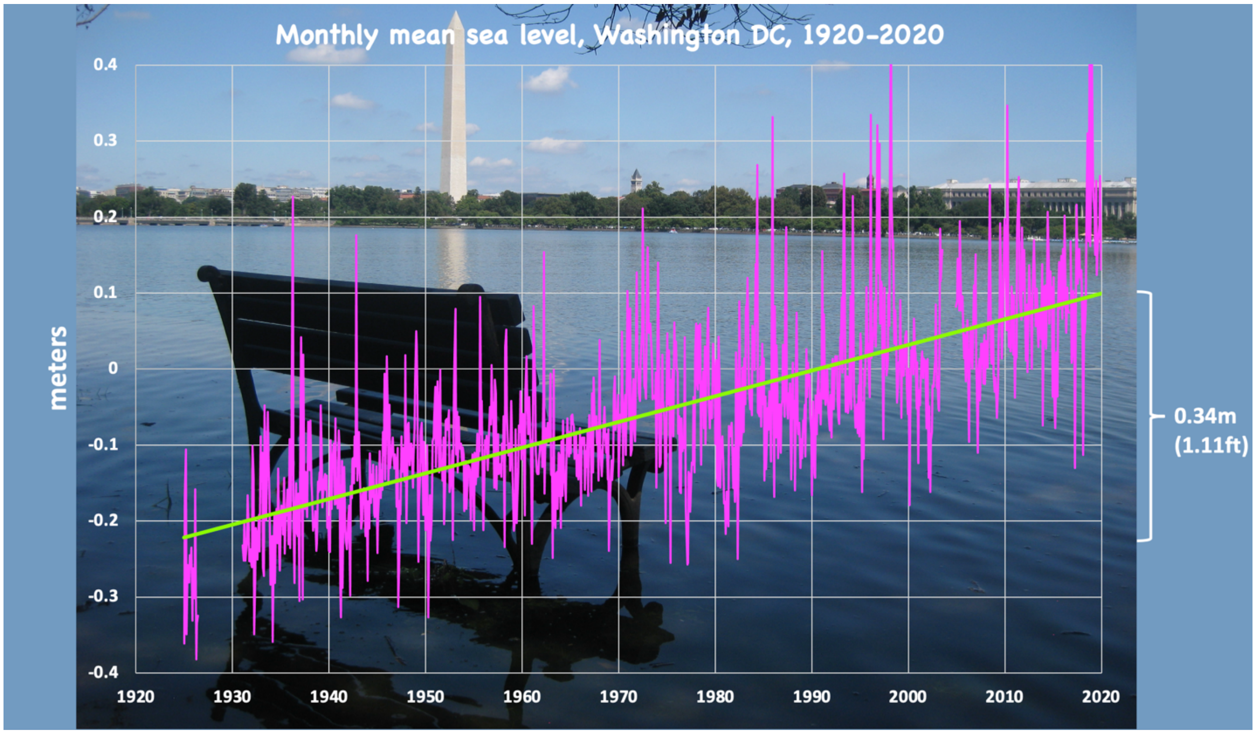
- Lowlying coastal zones are home to nearly one billion people
- The impact of severe storms is exacerbated by higher sea levels
- Infrequent coastal hazard events increase in frequency
- Sea level rise is not uniform everywhere. Ocean currents distribute heat unevenly, causing a larger rise in some regions. Similarly, some land surfaces are gradually subsiding (ex. delta regions) and are more vulnerable to rising sea level
- Rising sea level contaminates ground water supplies with salt water and kills salt-intolerant coastal vegetation
- Rising sea level impacts storm drains and wastewater systems in coastal urban areas
- Rising sea level exacerbates normal high tide impacts, especially seasonal "king tides," and recurrent El Nino events
- Hundreds of millions of people will be displaced by even small increases in global sea level
  (References: Climate.gov, https://www.climate.gov/news-features/understanding-climate/climate-change-global-sea-level; IPCC, https://www.ipcc.ch/srocc/chapter/summary-for-policymakers/)



Sea level changes are monitored globally in real time: https://tidesandcurrents.noaa.gov/sltrends/sltrends.html



On the Atlantic and Gulf coasts of the US sea level is increasing everywhere. The tide gauge in the US capitol, Washington DC, shows and increase of over a foot of sea level rise in the last century, which had led to daily flooding around several national monuments.



https://tidesandcurrents.noaa.gov/sltrends/sltrends_station.shtml?id=8594900